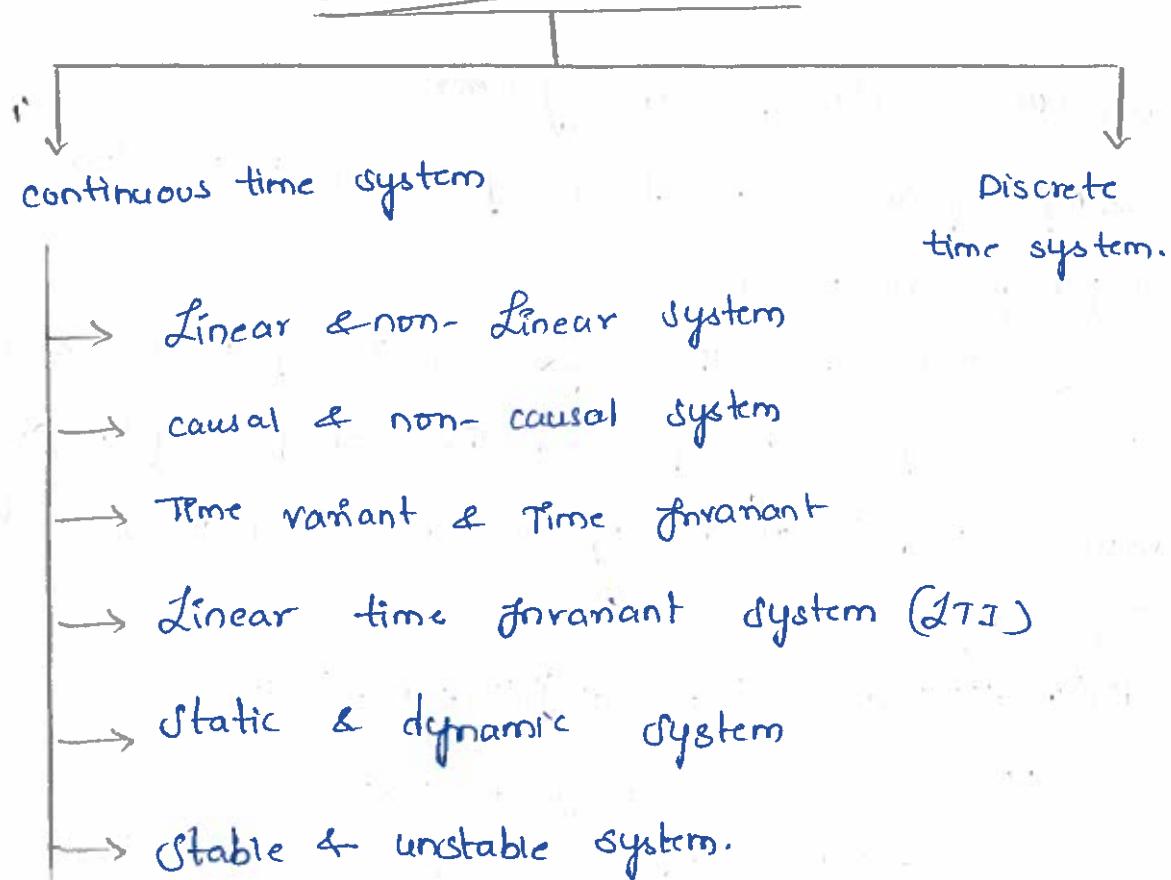


UNIT-3Signal Transmission through Linear System

System: → A set of elements or functional blocks

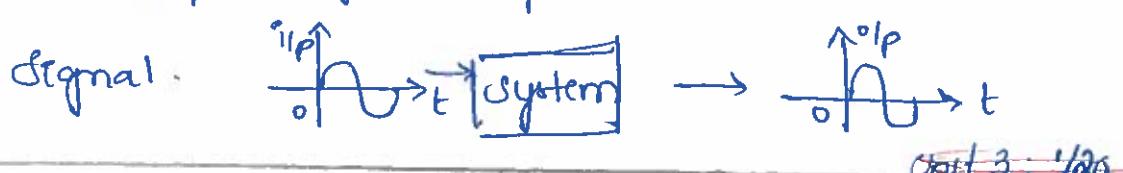
which provides output in response to an input.

→ system is a mathematically operator which maps input in to output.

Classification of System

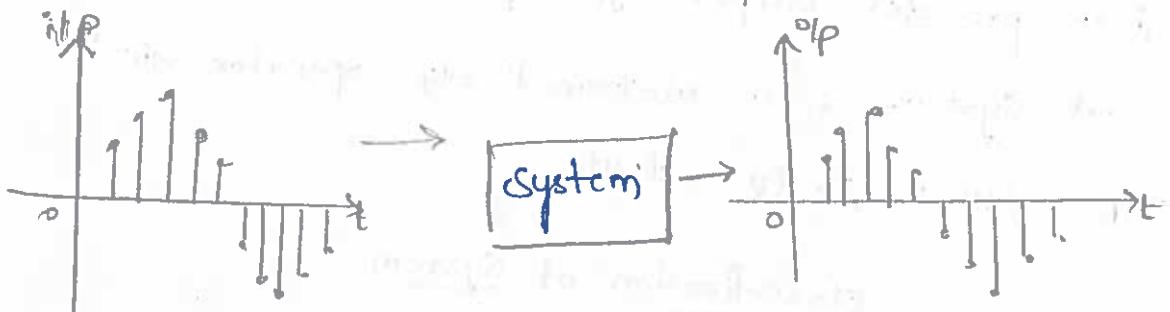
i) continuous time system:

→ continuous time system operates on a continuous time input signal to produce a continuous time output



ib Discrete time system:

A discrete time system operates an discrete time input signal to produce a discrete time output signal.



Linear & non-Linear System:

Any system can be linear system if it satisfies super position principle.

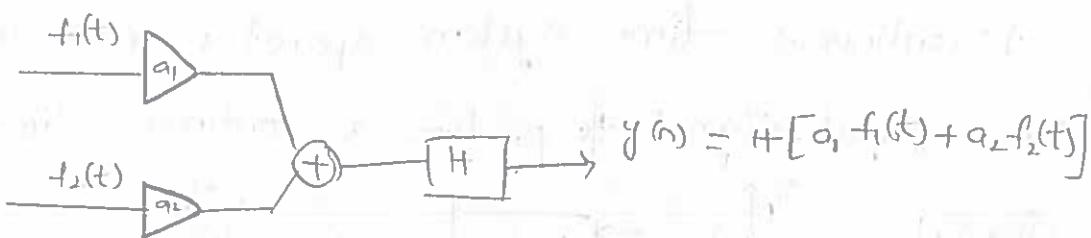
It states that the response of the system to a weighted sum of signals be equal to the corresponding weighted sum of outputs of the system to each of the individual input signal.

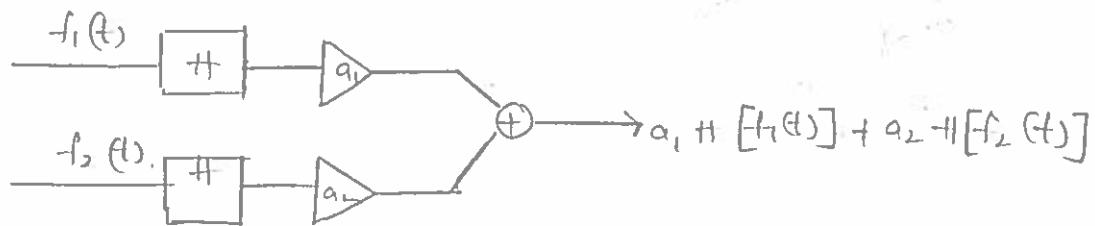
$$H[a_1 f_1(t) + a_2 f_2(t)] = a_1 H[f_1(t)] + a_2 H[f_2(t)]$$

Where a_1 & a_2 are weighted constants.

$$a_1 f_1(t) \xrightarrow{\text{Response}} a_1 H[f_1(t)]$$

$$a_2 f_2(t) \xrightarrow{\text{Response}} a_2 H[f_2(t)]$$





— Any system which does not obey superposition principle is said to be non-linear system.

Procedure for solving linear problems:

1, Apply different inputs separately and get the output.

2, Apply different inputs simultaneously & compute the output.

3, If both outputs are same it is linear else non-linear $y(t) = a \cdot x(t)$

$$\text{Step 1: } y_1(t) = a \cdot x_1(t).$$

$$y_2(t) = a \cdot x_2(t).$$

$$y(t) = a [x_1(t) + x_2(t)] \rightarrow \textcircled{1}$$

$$\text{Step 2: } y(t) = a x_1(t) + a x_2(t)$$

$$y(t) = a [x_1(t) + x_2(t)] \rightarrow \textcircled{2}$$

$$\textcircled{1} = \textcircled{2}$$

The given system is linear

2) $y(t) = x^2(t)$

$$\text{Step 1: } y_1(t) = a x_1^2(t)$$

$$y_2(t) = a x_2^2(t)$$

$$y(t) = a [x_1^2(t) + x_2^2(t)]$$

$$\text{Step 2: } y(t) = a [x_1(t) + x_2(t)]^2 \rightarrow \textcircled{2}$$

$\textcircled{1} \neq \textcircled{2}$

The system is non linear

$$3) y(t) = e^{x(t)}$$

$$\text{Step 1: } y_1(t) = e^{x_1 t}$$

$$y_2(t) = e^{x_2 t}$$

$$y(t) = e^{x_1 t} + e^{x_2 t} \rightarrow \textcircled{1}$$

$$\text{Step 2: } y(t) = e^{x_1(t)} + x_2(t) \rightarrow \textcircled{2} \text{ non linear}$$

$$4) y(t) = x(t^2)$$

$$\text{Step 1: } y_1(t) = x_1(t^2)$$

$$y_2(t) = x_2(t^2)$$

$$y(t) = x_1(t^2) + x_2(t^2) \rightarrow \textcircled{1}$$

$$\text{Step 2: } x_1(t^2) + x_2(t^2) \rightarrow \textcircled{2}$$

$$\textcircled{1} = \textcircled{2}$$

linear

$$5) y(t) = \cos[x(t)]$$

$$\text{Step 1: } y_1(t) = \cos[x_1(t)]$$

$$y_2(t) = \cos[x_2(t)]$$

$$y(t) = \cos x_1(t) + \cos x_2(t) \rightarrow \textcircled{1}$$

$$\text{Step 2: } \cos[x_1(t) + x_2(t)] \rightarrow \textcircled{2}$$

Causal & non-causal system:

A system is said to be causal if output depends only on present input, past input & past output.

A system is said to be non causal if the output depends on the future values of ⁱⁿ input.

$$1. \quad y(t) = [x(t+1) - x(t)]$$

$$= [x(t+1) - x(0)]$$

$$= [x(1) - x(0)]$$

non causal.

$$2. \quad y(t) = (t+3)x(t-3)$$

$$y(0) = (0+3)x(0-3)$$

$$y(0) = 3x(-3)$$

causal

$$3. \quad y(t) = x(t^2)$$

$$y(2) = x(4)$$

non causal.

$$4. \quad y(t) = x(t) - x(t-1)$$

$$y(1) = x(1) - x(0)$$

$y(1) = x$ causal

$$5. \quad y(t) = x(t) + \int_0^t x(\lambda) d\lambda.$$

$$y(t) = x(t) + \left[\frac{x^2}{2} \right]_0^t$$

$$y(t) = x(t) + \frac{x^2}{2} [t-0]$$

$$y(1) = x(1) + \frac{x^2}{2}(1) \quad \text{causal.}$$

Unit 3: 3/20

$$6) y(t) = 3n(t+3)$$

$$y(1) = 3n(1+3)$$

$$y(1) = 3n(4).$$

non causal.

Time Variant & Time Invariant

A system is said to be time invariant if the system does not depend on time.

i.e. system delay is not a function of time.

Procedure:

1, Shift the input only and get the output -

2, Shift the entire system & get the output.

3, If both steps are identical for output then it is time invariant system.

$$1) y(t) = 4n(t)$$

$$\delta_1 = y(t) = 4n(t-1)$$

$$\delta_2 = y(t-1) = 4n(t-1)$$

$\delta_1 \neq \delta_2$ Time variant.

$$2) y(t) = 4t n(t)$$

$$\delta_1 = y(t) = 4t n(t)$$

$$\delta_2 = y(t-1) = 4(t-1) n(t-1)$$

$\delta_1 \neq \delta_2$ Time variant.

$$3) y(t) = 5t [n(t)]^2$$

$$\delta_1 = y(t) = 5t [n(t-1)]^2$$

$$\delta_2 = y(t-1) = 5(t-1) [n(t-1)]^2$$

$\delta_1 \neq \delta_2$ Time variant.



$$y(t) = n(t+1) e^{-t}$$

∴ $s_1 = y(t) = n(t) e^{-t}$

$$s_2 = y(t-1) = n(t) e^{-(t-1)} \quad (e^{-1} \text{ is constant here})$$

$$s_1 = s_2$$

time invariant.

Linear Time Invariant (LTI) system:

— Any system which obeys the linearity & time invariant property is called a LTI (Linear time invariant system)

— Any system which obey linearity & does not obey time invariant property is called 'linear time variant system'.

i) $y(t) = a n(t).$

Linearity: $y_1(t) = a x_1(t) \quad \& \quad y_2(t) = a x_2(t)$

$$s_1 = y(t) = a [x_1(t) + x_2(t)]$$

$$s_2 = y(t) = a [x_1(t) + x_2(t)]$$

$$s_1 = s_2$$

Linear.

TIV: $s_1 = y(t) = a n_1(t-1)$

$$s_2 = y(t-1) = a n_1(t-1)$$

$$s_1 = s_2$$

Time invariant

∴ system is linear time invariant.

$$2) y(t) = t \cdot n(t)$$

S1 Linearity: $y_1(t_1) = t \cdot n_1(t_1)$

$$y_2(t_2) = t \cdot n_2(t_2)$$

$$s_1 = y(t) = t [n_1(t) + n_2(t)]$$

$$s_2 = y(t) = t [n_1(t) + n_2(t)]$$

$$s_1 = s_2$$

Linear

PIV: $s_1 = y(t) = t \cdot n(t-1)$

$$s_2 = y(t-1) = t-1 \cdot n(t-1)$$

$$s_1 \neq s_2$$

Time variant.

\therefore system is linear & time variant.

$$3) y(t) = a \cdot n(t) + b$$

S1 $y(t) = a \cdot n(t) + b$

$$y_1(t_1) = a \cdot n_1(t_1) + b$$

$$y_2(t_2) = a \cdot n_2(t_2) + b$$

$$s_1 = y(t) = a \cdot n_1(t) + a \cdot n_2(t) + b$$

$$s_2 = y(t) = a \cdot n_1(t) + a \cdot n_2(t) + b$$

$$s_1 = s_2$$

Linear.

PIV: $s_1 = y(t) = a \cdot n(t-1) + b$

$$s_2 = y(t-1) = a \cdot n(t-1) + b$$

$$s_1 = s_2$$

Time variant

Static & dynamic systems:

→ A static system is said to be static, if its output at any instant ~~for~~ depends only on the present values of the input. It is also known as 'Memory less system.'

$$y(t) = a x(t)$$

$$\text{at } t=0, \quad y(0) = a x(0)$$

$$\text{at } t=1, \quad y(1) = a x(1)$$

→ A system is said to be dynamic if its output depends on present and past values of the input.

Ex: $y(t) = x(t-1) + x(t-2) + x(t)$

$$\text{at } t=2$$

$$y(2) = x(2-1) + x(2-2) + x(2)$$

$$= x(1) + \underbrace{x(0)}_{\substack{\text{past value} \\ \text{of input}}} + x(2).$$

$\xrightarrow{\text{present value of the input}}$

It is also called as system with memory.

Impulse response:

The response of a system for an impulse input is called impulse response of the system and it is denoted as $h(t)$.

$$\delta(t) \rightarrow \boxed{\text{System}} \rightarrow h(t).$$

System is characterised by its impulse response

$$\text{-P/I P} \rightarrow \boxed{\text{System}} \rightarrow \text{O/P res}$$

Unit 3: 5/20

filter characteristics of a Linear System:

1) Ideal LPF (Low pass filter)

Ideal Low pass filters transmits all the signals below ideal certain frequency β Hz without any distortion.

The range of frequencies from δ to β Hz is called pass band of lowpass filter & the cut frequency β Hz is called the cut-off frequency of the ideal lowpass filter.

The transfer function of the ideal low pass filter can be written as

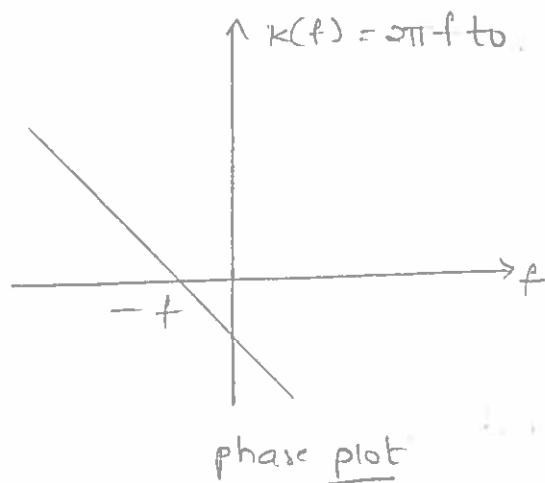
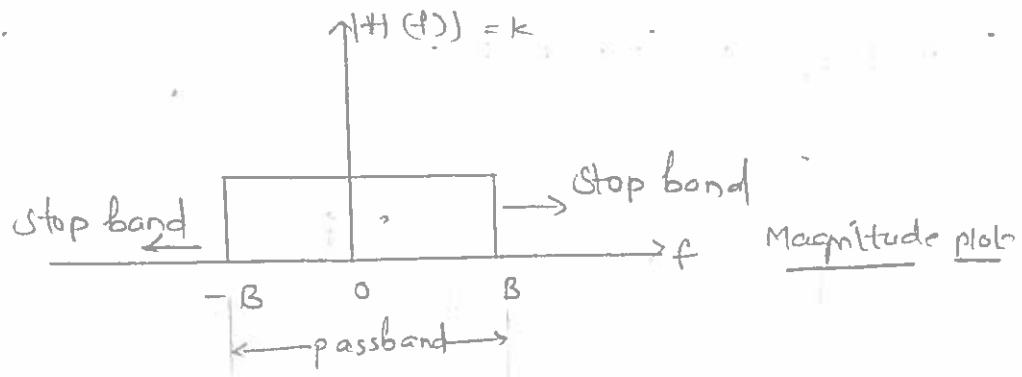
$$H(f) = \begin{cases} k e^{-j2\pi f t_0} ; & -B \leq f < B \\ 0 & ; |f| > B \end{cases}$$

Where k is amplitude & treated to be unity.

i.e. for $k=1$

$$H(f) = \begin{cases} e^{-j2\pi f t_0} ; & -B \leq f < B \\ 0 & ; |f| > B \end{cases}$$

By inverse fourier transform $h(t)$ can be obtained for ideal low pass filter.



Inverse FT is

$$h(t) = \int_{-\infty}^{\infty} H(f) e^{j2\pi ft} df; \quad \omega = 2\pi f$$

$$h(t) = \int_{-B}^{B} e^{-j2\pi ft_0} e^{j2\pi ft} df$$

$$h(t) = \int_{-B}^{B} e^{-j2\pi \beta (t_0 - t)} df$$

$$h(t) = \frac{1}{j2\pi(t-t_0)} \left[e^{-j2\pi f (t_0 - t)} \right]_{-B}^B$$

$$= \frac{1}{j2\pi(t-t_0)} \left[e^{j2\pi B(t-t_0)} - e^{-j2\pi B(t-t_0)} \right]$$

$$h(t) = \frac{1}{\pi(t-t_0)} \left[\frac{e^{j2\pi B(t-t_0)} - e^{-j2\pi B(t-t_0)}}{2j} \right]$$

Unit 3: 6/20

$$\frac{1}{\pi(t-t_0)} \sin[2\pi B(t-t_0)] \quad (\because \text{sinc})$$

$$-\omega_B \left[\frac{\sin 2\pi B(t-t_0)}{2\pi B(t-t_0)} \right] = 2\text{sinc}(\omega_B(t-t_0))$$

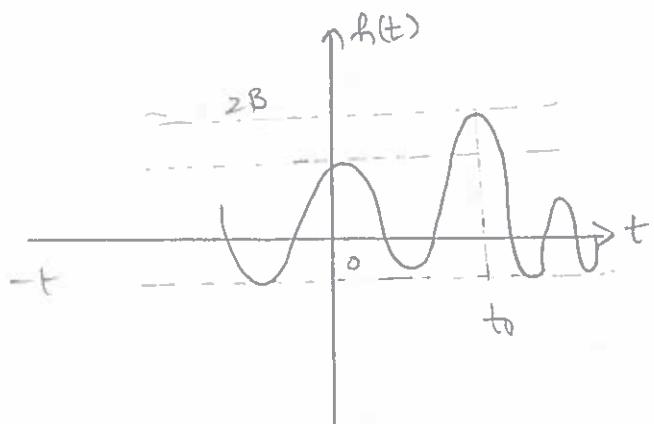
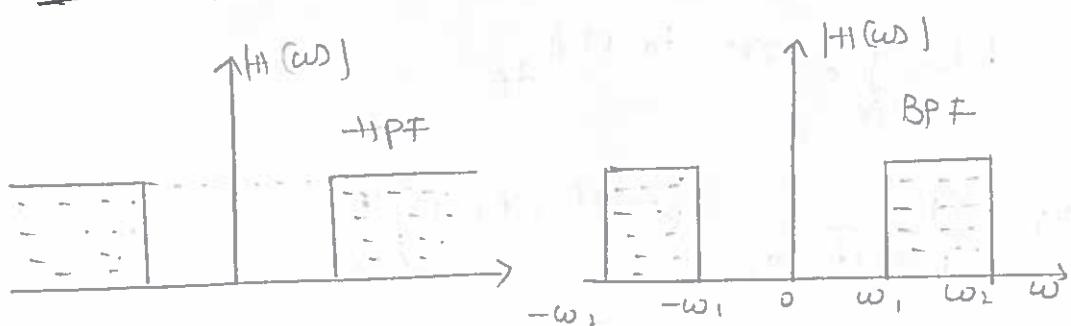


Figure shows that impulse response is 0 & also exist at negative values of t .

But Actually, unit impulse is applied at $t=0$ always exist at negative values of t .

practically it is impossible to such a system.

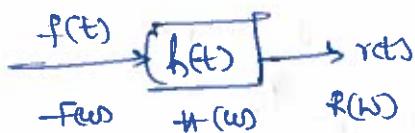
Ideal high pass & band pass filters



Note: In Realizability of Ideal Low pass filter its response begins before input is apply & hence it is not physically realized.

The frequency responses of HPF (High pass filter) BPF (Band pass filter) are shown above. These have sharp transition. In frequency response all ideal filters are physically not realizable. Since their impulse response is non-causal.

Introduction for filter characteristics:



$$r(w) = f(w) \cdot h(w).$$

The spectrum of output is $r(w) = f(w) \cdot h(w)$, i.e., the system acts as a kind of filter to various frequency components.

Sum frequency components are boosted in strength and some are attenuated and some remain unaffected. Similarly, each frequency component undergoes at different amount of phase shift.

i.e., modification is carried out according to $h(w)$.

Distortionless transmission through system:



It means that output signal is an exact replica of the input signal.

The difference between input and output of such systems is that,

- 1. Amplitude of the output signal may increase or decrease by some factor wrt output.
- 2. the output signal may be delayed in time wrt input signal because of system delay.
- 3. output signal $y(t)$ can be written in terms of input signal $x(t)$ as

$$y(t) = k \cdot x(t-t_0)$$

k = constant - change in amplitude

if $(t-t_0)$ = time delay in transmission

$$\mathcal{F}\{y(t)\} = \mathcal{F}\{k \cdot x(t-t_0)\}$$

$$Y(f) = \mathcal{F}\{k \cdot x(t-t_0)\}$$

From time shift property of Fourier transform

$$Y(f) = k \cdot X(f) e^{-j2\pi f t_0}$$

$$\frac{Y(f)}{X(f)} = k e^{-j2\pi f t_0}$$

Transfer function is given by

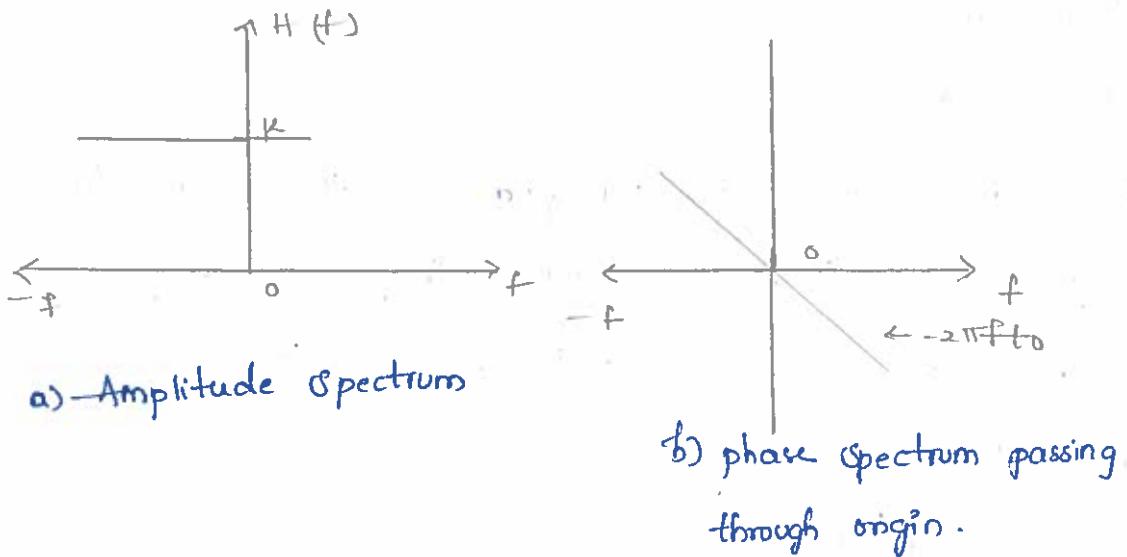
$$H(f) = \frac{Y(f)}{X(f)}$$

$$H(f) = k e^{-j2\pi f t_0}$$

Transfer function has constant amplitude & at all frequencies & phase shift is given by

$$\theta(f) = -2\pi f t_0$$

$$= (-2\pi t_0) f$$



Ex: consider a time domain signal as $x(t) = \cos(2\pi ft)$

Let the output signal be same in -Amplitude but shifted in time by t_0

$$\begin{aligned}
 y(t) &= \cos(2\pi f (t-t_0)) \\
 &= \cos(2\pi ft - 2\pi ft_0) \\
 &= \cos[2\pi ft - \phi(t)] \quad [\text{here } \phi(t) = -2\pi ft_0]
 \end{aligned}$$

\therefore phase shift of $y(t)$ is $\phi(t) = -2\pi ft_0$

which is proportional to frequency.

Types of Distortionless

1, Amplitude distortionless

2, phase distortion

Amplitude distortion: Amplitude distortion occurs when Magnitude $H(\omega)$ is not constant over frequency band of interest and the frequency components present in the input signal are transmitted with different gain and

Unit 3: 8/20

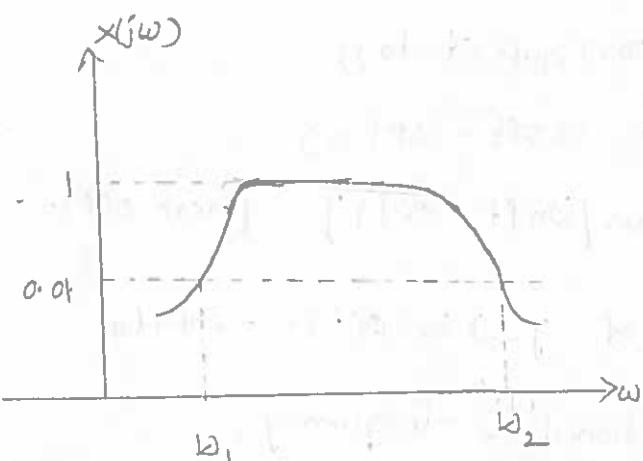
attenuation.

phase distortion:

phase distortion occurs when phase of $\text{H}(\omega)$ is not linearly changing with time & different frequency components in the input are subjected to different time delay during transmission.

Signal band width:

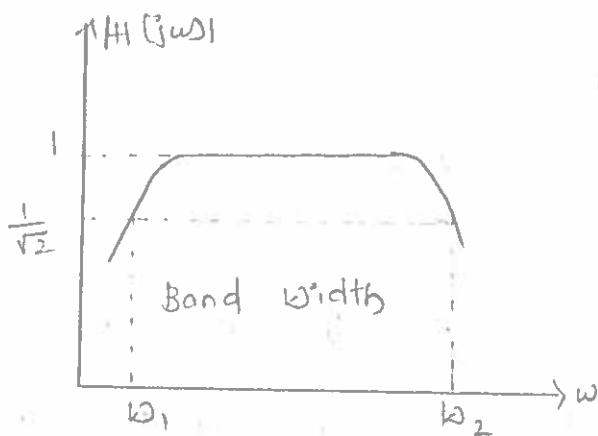
The band of frequencies that contains most of the signal energy is called band width of signal denoted by ~~BW~~. BW.



It is the range of significant signal frequencies which are present in the signal.

Observe the wave form, $x(t)$ has significant frequency from w_1 to w_2 & the band width of this signal is $w_2 - w_1$, and all the physically obtained signals have limited band width.

System band width



The Band width of the system is defined as range of frequencies over which Magnitude of $H(\omega)$ remains within $\frac{1}{\sqrt{2}}$ times of its mid band value.

For distortionless transmission the system must have infinite band width, but physical systems are limited to finite band width.

So, a system with finite band width can provide distortionless transmission from band limited signal.

If, magnitude of $H(\omega)$ remains constant over band width of signal.

The range of frequency for which $|H(j\omega)|$ of the system, remains within $\frac{1}{\sqrt{2}}$ of its maximum value.

Causal system:

$$If \quad n(t) = 0; \quad t < 0$$

$$n(t-t_0) = 0; \quad t < t_0.$$

That is if input is 0 for $t < t_0$ then output is also zero for $t < t_0$.

— Any system which does not obey the above rule is a non causal if two inputs to a causal system are equal up to some time t_0 then corresponding output must be equal up to that time constant.

paley weiner criterion

paley weiner criterion gives condition for causality in frequency domain or in other words the frequency domain equipment of causal system is $H(\omega)$ consider a system with transfer function $H(\omega)$ — the necessary and sufficient condition for $H(\omega)$ to be transfer function causal

— function is

$$H(\omega) = \int_{-\infty}^{\infty} \frac{\ln|H(j\omega)|}{1+\omega^2} d\omega < \infty \rightarrow ①$$

provided $|H(j\omega)|$ is square integrable.

$$H(\omega) = \int_{-\infty}^{\infty} |H(j\omega)|^2 d\omega < \infty \rightarrow ②$$

This is paley weiner criterion

If condition '2' is not satisfied then condition (1) is neither necessary nor sufficient.

physical / realizability:

→ system is said to be physically realizable if it obeys causal cond?

i.e. $h(t) = 0; t < 0$ time domin.

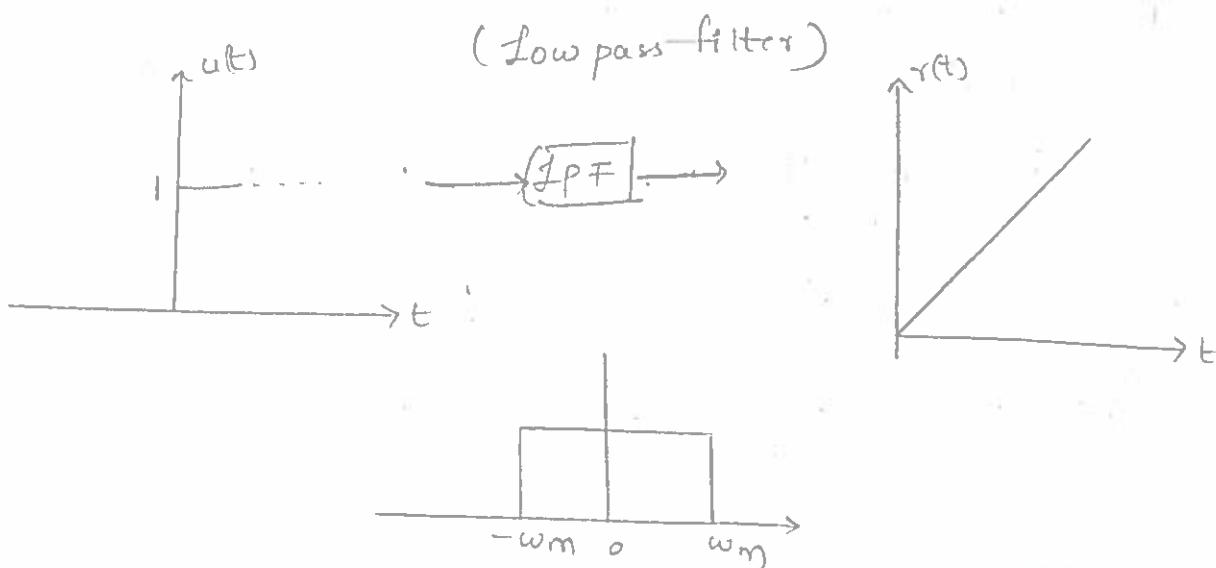
$$\text{ex: } H(j\omega) = \frac{1}{1+j\omega}$$

$$h(t) = e^{-t} u(t); t > 0 \\ = 0; t < 0$$

So, the above system for transfer function is realizable in frequency domain.

$$\int_{-\infty}^{\infty} \frac{|H(j\omega)|}{1+\omega^2} d\omega$$

Relation between rise time & band width



Unit 3: 10/20

- If a unit step function $u(t)$ is applied to an ideal Low pass filter the output will show a gradual rise instead of a sharp rise in the input.
- The rise time ' t_r ' is the time required by the response to reach its final value from initial value.
- The transfer function of ideal low pass filter is given by:

$$H(\omega) = |H(\omega)| e^{j\phi(\omega)}$$

$$= \epsilon(\omega) e^{-j\omega t_0}$$

→ rectangular pulse with width $\Delta\omega$.

for $-\beta \leq t \leq \beta$

$$\text{i.e., } -\omega_m \leq \omega \leq \omega_m; \quad \omega_m = 2\pi\beta.$$

$$\text{And } \delta(\omega) = -2\pi t_0 = -\omega t_0.$$

The Fourier transform of unit step function $u(t)$ is

$$\text{FT} \{ u(t) \} = \pi \delta(\omega) + \frac{1}{j\omega}.$$

Fourier transform of response $\delta(\omega)$ input $u(t) \otimes H(\omega)$ are related as.

$$F(\omega) = \left[\pi \delta(\omega) + \frac{1}{j\omega} \right] \quad H(\omega) = \pi \delta(\omega) \cdot H(\omega) + \frac{1}{j\omega} H(\omega).$$

$\delta(\omega)$ exists only for $\omega = 0$ & $H(\omega)/\omega_{z0} \approx 1$

$$\therefore F(\omega) = \pi \delta(\omega) + \frac{1}{j\omega} H(\omega)$$

($\because \delta(\omega) = \text{impulse function}$)

Taking inverse fourier transform for above eq'

$$r(t) = \text{IFT} \{ R(\omega) \} = \text{IFT} \{ \pi \delta(\omega) + \frac{1}{j\omega} + Q(\omega) \}$$

$$= \text{IFT} \{ \pi \delta(\omega) + \frac{1}{j\omega} Q(\omega) e^{-j\omega t_0} \}$$

$$= \text{IFT} \{ \pi \delta(\omega) \} + \text{IFT} \{ \frac{1}{j\omega} Q(\omega) e^{-j\omega t_0} \}$$

$$r(t) = l_2 + \text{IFT} \left\{ \frac{1}{j\omega} Q(\omega) e^{-j\omega t_0} \right\}$$

$$r(t) = l_2 + \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{1}{j\omega} Q(\omega) e^{-j\omega t_0} e^{j\omega t} d\omega$$

$$r(t) = l_2$$

$$\begin{bmatrix} \therefore 1 \xrightarrow{\text{FFT}} 2\pi \delta(\omega) \\ l_2 \xrightarrow{\text{IFT}} \pi \delta(\omega) \end{bmatrix}$$

i.e., $E_T(\omega) = 1$, for $-w_m \leq \omega \leq w_m$.

$$r(t) = l_2 + \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{e^{j\omega(t-t_0)}}{j\omega} d\omega$$

$$r = l_2 + \frac{1}{2\pi} \int_{-w_m}^{w_m} \frac{\cos \omega(t-t_0) + j \sin \omega(t-t_0)}{j\omega} d\omega.$$

$$(\because e^{j\theta} = \cos \theta + j \sin \theta)$$

$$= l_2 + \frac{1}{2\pi} \int_{-w_m}^{w_m} \frac{\cos \omega(t-t_0)}{j\omega} d\omega + \frac{1}{2\pi} \int_{-w_m}^{w_m} \frac{\sin \omega(t-t_0)}{\omega} d\omega$$

In the above eq' $\int_{-w_m}^{w_m} \frac{\cos \omega(t-t_0)}{j\omega} d\omega$ is odd function.

then it is equal to zero and

$$\int_{-w_m}^{w_m} \frac{\sin \omega(t-t_0)}{\omega} d\omega \text{ is Even}$$

$$r(t) = \frac{1}{2} + \frac{1}{\pi} \times 2 \int_0^{w_m} \frac{\sin \omega(t-t_0)}{\omega} d\omega$$

Unit 3: 14/20

$$= \frac{1}{2} + \frac{1}{2\pi} \times \left[\int_0^{\omega_m} \frac{\sin \omega(t-t_0)}{\omega} d\omega \right]$$

$$\left\{ \because \int_0^{\infty} \frac{\sin \omega}{\omega} d\omega = \frac{\pi}{2} \right\}$$

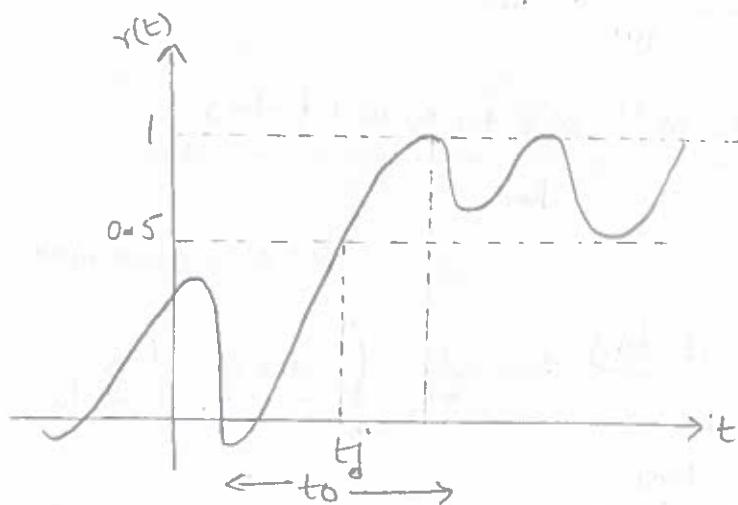
$$r(t) = \frac{1}{2} + \frac{1}{\pi} [\sin \omega_m(t-t_0)]_{0}^{\omega_m}$$

$$r(t) = \frac{1}{2} + \frac{1}{\pi} \sin \omega_m(t-t_0) \rightarrow \text{sinc integral}$$

The $r(t)$ is time given as,

$$tr = \frac{2\pi}{\omega_m} = \frac{1}{B} \\ \rightarrow \text{cut off frequency of LPF.}$$

$$\frac{1}{tr} = \frac{\omega_m}{2\pi} = B.$$



$t \rightarrow \infty; r(t) \approx 1$

$t \rightarrow -\infty; r(t) \approx 0$

12

The impulse response of continuous time signal is, given as -

$h(t) = \frac{1}{RC} e^{-t/RC} u(t)$ determine the frequency response & plot the magnitude & phase plots.

Sol consider \mathcal{F} of $h(t)$ is given by

$$\begin{aligned} H(\omega) &= \int_{-\infty}^{\infty} h(t) e^{-j\omega t} dt \\ &= \int_{-\infty}^{\infty} \frac{1}{RC} e^{-t/RC} e^{-j\omega t} u(t) dt \end{aligned}$$

$$\text{But } u(t) = \begin{cases} 1; & t \geq 0 \\ 0; & t < 0 \end{cases}$$

$$H(\omega) = \frac{1}{RC} \int_0^{\infty} e^{-t/RC} e^{-j\omega t} dt$$

$$H(\omega) = \frac{1}{RC} \int_0^{\infty} e^{-t} \left(\frac{1}{RC} + j\omega \right) dt$$

$$= \frac{1}{RC} \left[\frac{-1}{(j\omega + 1/RC)} e^{-t(j\omega + 1/RC)} \right]_0^{\infty}$$

$$= \frac{1}{RC} \left[\frac{-1}{(j\omega + 1/RC)} (e^{j\omega} - e^0) \right]$$

$$= \frac{1}{RC} \times \frac{-1}{(j\omega + 1/RC)} [0-1]$$

$$= \frac{1}{RC} \times \frac{1}{j\omega + 1/RC} = \frac{1}{1+j\omega RC}$$

$$\boxed{H(\omega) = \frac{1}{1+j\omega RC}}$$

Unit 3: 12/20

Unit - 3, Pg - 23/40

To plot magnitude & phase

$$H(\omega) = \frac{1}{1+j\omega RC} \times \frac{1-j\omega RC}{1-j\omega RC}$$

$$H(\omega) = \frac{1-j\omega RC}{1-(j\omega RC)^2}$$

$$H(\omega) = \frac{(1-j\omega RC)}{1+(\omega RC)^2}$$

$$H(\omega) = \frac{1}{1+(\omega RC)^2} + \frac{j(\omega RC)}{1+(\omega RC)^2}$$

To plot magnitude

$$|H(\omega)| = \sqrt{\left(\frac{1}{1+(\omega RC)^2}\right)^2 + \left[\frac{(1-\omega RC)}{1+(\omega RC)^2}\right]^2}$$

$$|H(\omega)| = \sqrt{\frac{1+(\omega RC)^2}{(1+(\omega RC)^2)^2}}$$

$$|H(\omega)| = \frac{1}{\sqrt{1+\omega^2}}$$

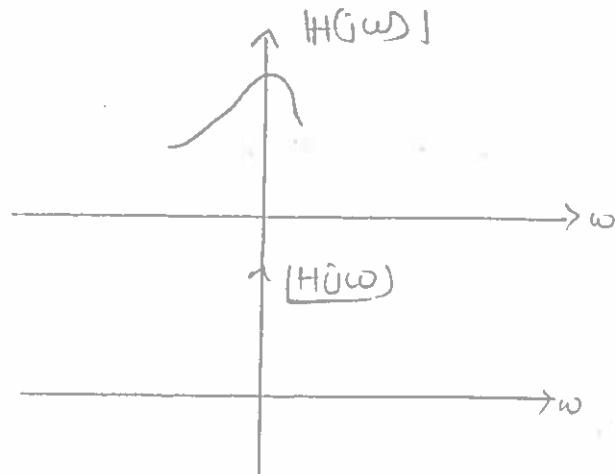
$$j + RC = 1$$

$$|H(\omega)| = \frac{1}{\sqrt{1+\omega^2}}$$

phase ϕ

$$\angle H(\omega) = \tan^{-1} \left[\frac{\omega RC / 1+(\omega RC)^2}{1/1+(\omega RC)^2} \right]$$

$$\begin{aligned}|H(\omega)| &= -\tan^{-1}(-\omega R/C) \\ &= -\tan^{-1}(-\omega)\end{aligned}$$

Ans

For the system shown find the Fourier transform & impulse function.

$$f(t) = \begin{cases} e^{-at}, t > 0 \\ 0, t < 0 \end{cases}; Y(\omega) = \frac{1}{\alpha + j\omega}.$$

Q1 Fourier transform $F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$

$$F(\omega) = \int_0^{\infty} e^{-at} e^{-j\omega t} dt$$

$$= \int_0^{\infty} e^{-(a+j\omega)t} dt$$

$$= \left[\frac{e^{-(a+j\omega)t}}{-(a+j\omega)} \right]_0^{\infty}$$

$$F(\omega) = \frac{1}{-(a+j\omega)}$$

Unit 3: 13/20

Impulse response:

$H(\omega) = \frac{\text{Input}}{\text{Output}} \rightarrow \text{transfer function}$

$$f(\omega) \xrightarrow{\quad} \boxed{R(\omega) = Y(\omega)} \xrightarrow{\quad} H(\omega)$$

$$\frac{Y(\omega)}{f(\omega)} = \frac{\frac{1}{(\alpha+j\omega)}}{\frac{1}{(\alpha+j\omega)}}$$

$$H(\omega) = \frac{\alpha+j\omega}{\alpha+j\omega}$$

$$H(\omega) = \frac{\alpha+\alpha-\alpha+j\omega}{\alpha+j\omega}$$

$$H(\omega) = \frac{\alpha-\alpha}{\alpha+j\omega} - \frac{\alpha+j\omega}{\alpha+j\omega}$$

$$H(\omega) = \frac{\alpha-\alpha}{\alpha+j\omega} + 1$$

IFT is given by

$$\text{IFT } [H(\omega)] = k(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} H(\omega) e^{j\omega t} dt$$

$$= \frac{1}{2\pi} \int_0^{\infty} \left(\frac{\alpha-\alpha}{\alpha+j\omega} + 1 \right) e^{j\omega t} dt$$

$$= \frac{\alpha-\alpha}{2\pi} \left[\int_0^{\infty} \frac{1}{\alpha+j\omega} e^{j\omega t} dt \right] + \left[\int_0^{\infty} 1 \cdot e^{j\omega t} dt \right] \frac{1}{2\pi}$$

(14)

$$\left[\begin{array}{ccc} \because e^{-at} & \xleftrightarrow{\text{FT}} & \frac{1}{a+j\omega} \\ & & \\ \frac{1}{a+j\omega} & \xleftrightarrow{\text{IFT}} & e^{-at} \end{array} \right]$$

$$= \frac{a-\alpha}{2\pi} \left[e^{-at} u(t) + 2\pi \delta(\omega) \right] \left[\because \frac{1}{a+j\omega} \xleftrightarrow{\text{IFT}} e^{-at} \right]$$

$$f(t) = \frac{a-\alpha}{2\pi} \left[e^{-at} u(t) + \delta(\omega) \right]$$

$$\left[\because \int_{-\infty}^{\infty} e^{j\omega t} dt = 2\pi \delta(\omega) \right]$$

$$1 \xleftrightarrow{\text{IFT}} 2\pi \delta(\omega)$$

A system produces an output of $y(t) = e^{-t} u(t)$ for an input of $x(t) = e^{-2t} u(t)$ determine the impulse response & frequency response of the system.

$$y(t) = e^{-t} u(t); \quad x(t) = e^{-2t} u(t)$$

$$H(\omega) \xleftrightarrow{\text{FT}} h(t) = ?$$

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} \rightarrow \textcircled{1}$$

$$\text{FT } Y(\omega) = \int_{-\infty}^{\infty} y(t) e^{-j\omega t} dt$$

$$= \int_0^{\infty} e^{-2t} u(t) e^{-j\omega t} dt$$

$$= \int_0^{\infty} e^{-(2+j\omega)t} dt$$

Unit 3: 14/20

$$= \left[\frac{e^{-(2+j\omega)t}}{e^{-2+j\omega}} \right]_0^\infty$$

$$x(t) = \frac{1}{-(2+j\omega)} (e^\infty + e^0)$$

$$x(\omega) = \frac{1}{2+j\omega}$$

$$y(\omega) = \int_0^\infty e^{-t} u(t) e^{-j\omega t} dt$$

$$= \int_0^\infty e^{(1+j\omega)t} dt$$

$$= \left[\frac{e^{-(1+j\omega)t}}{-1-j\omega} \right]_0^\infty$$

$$y(\omega) = \frac{1}{-1-j\omega} (e^\infty - e^0)$$

$$Y(\omega) = \frac{1}{1+j\omega}$$

$$\therefore 0 \Rightarrow H(j\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{\frac{1}{1+j\omega}}{\frac{1}{2+j\omega}}$$

$$(H(j\omega) = \frac{2+j\omega}{1+j\omega})$$

$$= \frac{1+j\omega+1}{1+j\omega}$$

$$= 1 + \frac{1}{1+j\omega}$$

$$= \delta(t) + e^{-t} u(t)$$

$$u(t) = 1$$

$\frac{1}{1+j\omega}$ is in form of $\frac{1}{a+j\omega}$

$$e^{-at} \delta(t) = 1$$

$$\text{IFT } \{H(\omega)\} = h(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} H(\omega) e^{j\omega t} dt$$

$$h(t) = \frac{1}{2\pi} \int_0^{\infty} \frac{2+j\omega}{1+j\omega} e^{j\omega t} d\omega$$

$$h(t) = \frac{2+j\omega}{2\pi} e^{-t}$$

The linear system impulse response is $[e^{-2t} + e^{-3t}] u(t)$

find the excitation to produce an output of $t e^{-2t} u(t)$.

(Q) Given $h(t) = [e^{-2t} + e^{-3t}] u(t)$

$$H(\omega) = \frac{1}{2+j\omega} + \frac{1}{3+j\omega} \quad \left[\because h(t) = e^{-at} \xrightarrow{\text{IFT}} \frac{1}{a+j\omega} \right]$$

$$h(t) = e^{-2t} u(t) + e^{-3t} u(t) = h_1(t) + h_2(t).$$

$$H(\omega) = \int_{-\infty}^{\infty} h(t) e^{-j\omega t} dt$$

$$= \int_{-\infty}^{\infty} e^{-2t} u(t) e^{-j\omega t} dt$$

$$\text{but } u(t) = \begin{cases} 1; & t \geq 0 \\ 0; & t < 0 \end{cases}$$

$$H(\omega) = \int_0^{\infty} e^{-2t} e^{-j\omega t} dt$$

$$= \int_0^\infty e^{-(2+j\omega)t} dt$$

$$h_1(\omega) = \frac{-1}{(2+j\omega)} \left[e^{-(2+j\omega)t} \right]_0^\infty$$

$$= \frac{1}{-(2+j\omega)} (0-1)$$

$$\boxed{h_1(\omega) = \frac{1}{2+j\omega}}$$

$$h_2(\omega) = \int_0^\infty e^{-3t} u(t)$$

$$= \int_0^\infty e^{-3t} e^{-j\omega t} dt$$

$$= \int_0^\infty e^{-(3+j\omega)t} dt$$

$$h_2(\omega) = \frac{-1}{(3+j\omega)} \left[e^{-(3+j\omega)t} \right]_0^\infty$$

$$= \frac{-1}{(3+j\omega)} (0-1)$$

$$\boxed{h_2(\omega) = \frac{1}{3+j\omega}} \Rightarrow R(t) = h(t) + h_2(t)$$

$$H(\omega) = \frac{1}{2+j\omega} + \frac{1}{3+j\omega}$$

$$r(t) = t \cdot e^{-2t} u(t)$$

$$R(\omega) = \int_{-\infty}^\infty r(t) e^{-j\omega t} dt$$

$$= \int_{-\infty}^\infty t e^{-2t} u(t) e^{-j\omega t} dt$$

$$= \int_0^\infty t e^{-2t} e^{-j\omega t} dt$$

$$= \int_0^\infty t e^{-(2+j\omega)t} dt$$

$$R(\omega) = \left[e^{-j(2+j\omega)t} \left[\frac{t}{(2+j\omega)} + \frac{1}{(2+j\omega)^2} \right] \right]_0^\infty$$

$$\left[\because \text{near } \omega = 0 \text{ and } \frac{1}{(2+j\omega)^2} \ll \frac{1}{(2+j\omega)} \right]$$

$$R(\omega) = -1 \left[0 - \frac{1}{(2+j\omega)^2} \right]$$

$$R(\omega) = \frac{1}{(2+j\omega)^2}$$

$$H(\omega) = \frac{R(\omega)}{X(\omega)}$$

$$\left[H(\omega) = \frac{R(\omega)}{X(\omega)} \right] = \frac{\frac{1}{(2+j\omega)^2}}{\frac{1}{(2+j\omega)} + \frac{1}{(3+j\omega)}}$$

$$X(\omega) = \frac{\frac{1}{(2+j\omega)^2}}{\frac{3+j\omega + 2+j\omega}{(2+j\omega)(3+j\omega)}}$$

$$X(\omega) = \frac{(2+j\omega)(3+j\omega)}{(2+j\omega)^2}$$

$$= \frac{5+j\omega}{(2+j\omega)^2}$$

$$X(\omega) = \frac{3+j\omega}{(5+j\omega)(2+j\omega)}$$

Unit-3: 16/20

Using practical fraction expansions

$$\frac{3+j\omega}{(5+j\omega)(2+j\omega)} = \frac{A}{(5+j\omega)} + \frac{B}{(2+j\omega)}$$

$$3+j\omega = A(2+j\omega) + B(5+j\omega)$$

$$= 2A + j\omega A + 5B + 2Bj\omega$$

$$= 2A + 5B + j\omega(A+2B)$$

$$\text{put } j\omega = 0$$

$$2A + 5B = 3 \times 1 \Rightarrow 2A + 5B = 3$$

$$A + 2B = 1 \times 2 \Rightarrow \begin{array}{rcl} 2A + 4B & = & 2 \\ - & - & - \\ 0 + B & = & 1 \end{array}$$

(B=1)

$$\text{Sub } j\omega + 2 = 1$$

$$\boxed{A=-1}$$

$$R(\omega) = \frac{-1}{2(5+j\omega)} + \frac{1}{(2+j\omega)} \quad \left[\begin{array}{l} y(\omega) \xleftrightarrow{\text{FI}} r(t) \\ = e^{-st} u(t) \end{array} \right]$$

$$r(t) = \left[-\frac{1}{2} e^{-5t} + e^{-2t} \right] u(t)$$

Differential Equation:

The linear constant coefficient differential eqn is given

by.

$$\sum_{k=0}^N a_k \frac{d^k y}{dt^k} \quad y(t) = \sum_{k=0}^M b_k \frac{d^k x}{dt^k} \quad \xrightarrow{\text{D}}$$

Eqn ① gives the relation between input & output

To obtain frequency response & impulse response.

Time differential property of Fourier transform is given by

$$\frac{d}{dt} x(t) \xleftrightarrow{\text{FT}} j\omega x(\omega)$$

$$\Rightarrow \sum_{k=0}^N a_k(j\omega)^k y(\omega) = \sum_{k=0}^M b_k(j\omega)^k x(\omega)$$

$$\therefore \frac{y(\omega)}{x(\omega)} = \frac{\sum_{k=0}^M b_k(j\omega)^k}{\sum_{k=0}^N a_k(j\omega)^k}$$

But $\frac{y(\omega)}{x(\omega)} = H(\omega) \rightarrow$ Transfer function

$$H(\omega) = \frac{y(\omega)}{x(\omega)} = \frac{\sum_{k=0}^M b_k(j\omega)^k}{\sum_{k=0}^N a_k(j\omega)^k}$$

The differential eqn of system is given as $\frac{d^2 y(t)}{dt^2} +$

$$5 \frac{dy(t)}{dt} + 6y(t) = - \frac{d x(t)}{dt}$$

determine the frequency response & impulse response.

[Q]

Given:

$$\frac{d^2 y(t)}{dt^2} + 5 \frac{dy(t)}{dt} + 6y(t) = - \frac{d x(t)}{dt}$$

Taking FT on both sides.

Unit 3: 17/20

$H(\omega)$ = frequency response

$h(t)$ = impulse response.

$$Y(j\omega)^2 \propto \gamma(\omega) + 5j\omega \gamma(\omega) + 6\gamma(\omega)$$

$$= -j\omega x(\omega)$$

$$\Rightarrow \gamma(\omega) [G(\omega)^2 + 5j\omega + 6] = -j\omega x(\omega)$$

$$\frac{y(\omega)}{x(\omega)} = H(\omega) = \frac{-j\omega}{G(\omega)^2 + 5j\omega + 6}$$

$$H(\omega) = \frac{-j\omega}{(2+j\omega)(3+j\omega)}$$

$$\frac{-j\omega}{(2+j\omega)(3+j\omega)} = \frac{A}{2+j\omega} + \frac{B}{3+j\omega}$$

$$\frac{-j\omega}{G(\omega)^2 + j\omega + 6} = \frac{1}{G(\omega)^2 + 2j\omega + 3j\omega + 6}$$

$$-j\omega = A(3+j\omega) + B(2+j\omega)$$

$$-3A + 2B = 0$$

$$-A + B = -1$$

$$3A + 2B = 0$$

$$-A + B = -1$$

$$\underline{\underline{-A + 0 = 2}}$$

$$\boxed{A=2} \text{ sub } 2+B=-1$$

$$B = -1 - 2$$

$$\boxed{B=-3}$$

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{2}{(2+j\omega)} - \frac{3}{(3+j\omega)}$$

This is frequency response

Inverse Fourier transform of $H(\omega)$

$$h(t) = [2e^{-2t} - 3e^{-3t}] u(t)$$

$$\left[\therefore h(t) = e^{-at} \xleftrightarrow{\text{IFT}} \frac{1}{a+j\omega} \right]$$

The input voltage to the RC circuit is given by

$x(t) = t e^{-t/\tau_{RC}} u(t)$ & impulse response of this circuit is

given by, $h(t) = Y_{RC} e^{-t/\tau_{RC}} u(t)$. Find the output $y(t)$.

(Q): $x(t) = t e^{-t/\tau_{RC}} u(t)$

$$h(t) = \frac{1}{\tau_{RC}} e^{-t/\tau_{RC}} u(t).$$

$$y(t) = x(t) * h(t) \rightarrow \text{TD (Time domain)}$$

$$Y(\omega) = X(\omega) H(\omega) \rightarrow \text{FD (Frequency domain)}$$

$$x(t) = t e^{-t/\tau_{RC}} u(t)$$

Applying FT on both sides

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$= \int_{-\infty}^{\infty} t \cdot e^{-t/\tau_{RC}} u(t) e^{-j\omega t} dt$$

$$= \int_0^{\infty} t \cdot e^{-(V_{RC} + j\omega)t} dt$$

Unit 3: 10/20

$$x(\omega) = \left\{ e^{-(V_{RC} + j\omega)t} + \frac{1}{(V_{RC} + j\omega)^2} \right\}_0^\infty$$

$$= \frac{1}{1 + \left(\frac{j\omega V_{RC}}{V_{RC}}\right)^2}$$

$$x(\omega) = \frac{(V_{RC})^2}{(1 + j\omega V_{RC})^2}$$

$$h(t) = \frac{1}{V_{RC}} e^{-t/V_{RC}} u(t)$$

$$H(\omega) = \int_{-\infty}^{\infty} h(t) e^{-j\omega t} dt$$

$$H(\omega) = \frac{1}{V_{RC}} \int_0^{\infty} e^{-(V_{RC} + j\omega)t} dt$$

$$H(\omega) = \frac{1}{V_{RC}} \left\{ \frac{-1}{(V_{RC} + j\omega)} e^{-(V_{RC} + j\omega)t} \right\}_0^\infty$$

$$= \frac{1}{V_{RC}} \left[\frac{V_{RC}}{1 + j\omega V_{RC}} \right]$$

$$H(\omega) = \frac{1}{1 + j\omega V_{RC}}$$

$$Y(\omega) = \frac{(V_{RC})^2}{(1 + j\omega V_{RC})^2} - \frac{1}{(1 + j\omega V_{RC})}$$

$$Y(\omega) = \frac{(V_{RC})^2}{(1 + j\omega V_{RC})^3}$$

(19)

$$y(t) = \text{IFT}(Y(\omega))$$

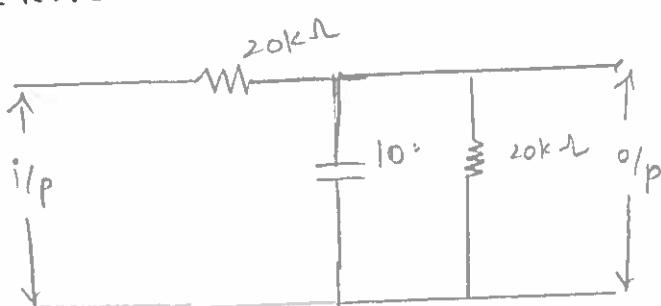
$$Y(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} Y(\omega) e^{j\omega t} dt$$

$$= \frac{1}{2\pi} \int_0^{\infty} \frac{(RC)^2}{(1+j\omega RC)^3} e^{j\omega t} dt$$

$$\left[\begin{array}{l} \because e^{-at} \xrightarrow{\text{FT}} \frac{1}{(a+j\omega)} \\ t \cdot e^{-at} \xrightarrow{\text{FT}} \frac{1}{(a+j\omega)^2} \\ t^2 e^{-at} \xrightarrow{\text{FT}} \frac{1}{(a+j\omega)^3} \end{array} \right]$$

$$\left[\frac{1}{RC} \quad \frac{t^2 e^{-t/RC}}{2\pi} u(t) \right]$$

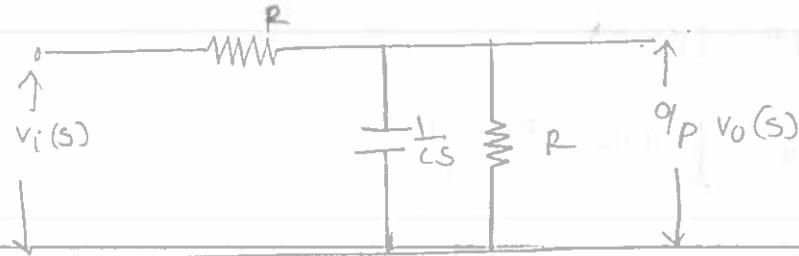
Determine maximum band width of signal that can be transmitted through a low pass LC filter shown in the figure. If over this band width the gain variation is to be 10% the phase variation is to be 7% of ideal characteristics.



59

The LC network transformed into s-domain representation.

Unit 3. 19/20



$$H(s) = \frac{V_o(s)}{V_i(s)}$$

$$= \frac{(R \parallel \frac{1}{Cs})}{(R + (R \parallel \frac{1}{Cs}))}$$

$$= \frac{\frac{R}{Cs}}{R + \frac{1}{Cs}} \cdot \frac{1}{R + (\frac{R}{Cs} / R + \frac{1}{Cs})}$$

$$= \frac{\frac{R}{(Rcs+1)}}{R + \frac{Rcs}{(Rcs+1)Cs}}$$

$$= \frac{\frac{R}{Rcs+1}}{R(\frac{R}{Rcs+1}) + \frac{R}{Cs}}$$

$$H(s) = \frac{1}{2 + SCR}$$

But $R = 20k\Omega$ & $C = 10\text{fF}$

$$H(s) = \frac{1}{2 + s(10 \times 10^{-9} \times 20 \times 10^3)}$$

$$\text{numerator } \frac{\frac{R}{Cs}}{R + \frac{1}{Cs}} = \frac{Rcs}{Rcs + 1}$$

$$\text{denominator } R + \left[\frac{\frac{R}{Cs}}{(R + \frac{1}{Cs})} \right]$$

$$= \frac{\frac{R}{Cs}}{Rcs + 1} = \frac{Rcs}{(Rcs + 1)Cs}$$

$$= \frac{R}{Rcs + 1} + R$$

20

$$= \frac{1}{2+2 \times 10^{-4} s}$$

$$= \frac{10^4}{2+2 \times 10^{-4} s}$$

$$H(s) = \frac{10000}{2s + 20000}$$

put $s = j\omega$

$$H(j\omega) = \frac{10000}{2j\omega + 20000} \rightarrow \textcircled{1}$$

$$|H(j\omega)| = \frac{1}{\sqrt{(20 \times 10^3)^2 + (\omega)^2}}$$

$$\angle H(j\omega) = \tan^{-1} \left(\frac{\omega}{20000} \right)$$

$$\textcircled{1} \Rightarrow \text{at } \omega = 0, |H(j\omega)|_{\omega=0} = \frac{10000}{20000} = 0.5$$

band width = 0.5

but there is 10^{-1} variation in gain over band width

$$H(\omega) = 0.5 - 0.5 \times 10^{-1}$$

$$= 0.5 - 0.05$$

$$= 0.45$$

$$|H(j\omega)| = \frac{10000}{\sqrt{B^2 + (20000)^2}}$$

~~Unit 3: 20/68~~

$$B^2 + (20 \times 10^3)^2 = \left(\frac{10000}{0.45} \right)^2$$

$$B^2 = \left(\frac{10000}{0.45} \right)^2 - \frac{1}{(20 \times 10^3)^2}$$

$\frac{1}{6}$